Wittgenstein and Turing on the Diagonal Argument

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Abstract: Philosophy of mathematics plays a crucial, if not a central, role throughout Wittgenstein's philosophy. Investigating Wittgenstein's interactions with his contemporary mathematicians is important in understanding the genesis of Wittgenstein's philosophy of mathematics and appreciating its significance. Juliet Floyd has thoroughly examined the personal and intellectual interactions between Wittgenstein and Turing (e.g., Floyd 2012; 2020). One notable example Floyd (2012) provides is what she calls "Wittgenstein's diagonal argument", presented in Wittgenstein's 1947 remark "A Variant of Cantor's Diagonal Proof" (RPP I: §1097). She argues that Wittgenstein's diagonal argument is influenced by Turing's 1936 paper "On Computable Numbers" (Turing 1936).

Although Floyd's comparison between Wittgenstein's and Turing's diagonal argument is illuminating, I shall argue that she misidentifies where Turing's influence on Wittgenstein lies. In this talk, I will first briefly survey what we know about Wittgenstein-Turing interactions by examining available historical documents. Second, I will give a tour of Turing's peculiar use of the diagonal argument in proving the undecidability results in his 1936 paper. After setting the stage, I will explain Floyd's reconstruction of Wittgenstein's 1947 variation of the diagonal argument and her account of Turing's influence on Wittgenstein. As I will argue, Floyd's account is based on a misleading interpretation of Turing's use of the diagonal argument in "On Computable Numbers". At the end of my talk, I will investigate in what sense and to what extent we can say Wittgenstein's 1947 variation of the diagonal argument is influenced by Turing.

References:

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